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A computerized lot-sizing rule for total operating cost optimization with pre assigned number of lots

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Abstract

A proper lot sizing is always viewed as a major problem in the context of buyer-supplier relationships. Several heuristics like Silver-Meal, LUC, PPB have been developed in this field as an aid to fix the proper lot size in order to minimize the total operating cost. However, they all suffer from various limitations and do well only under certain conditions.

The problem is none of them consider the variation of parameter like unit production cost as it is quite practical that raw material cost may vary from time to time which may affect unit production cost. That way most of the heuristics are rigid and lack flexibility in handling operating conditions beyond their scope.

However, the exact solution in more general situations, assuming integral-valued demand, is obtainable by Dynamic Programming based Wagner-Whitin algorithm which considers constraints like variation in unit production cost, limitations in production capacity or storage capacity is more flexible compared to other heuristics and ensures optimum solution. Furthermore, in practical production scenario appearance of few more constraints cannot be ruled out. For example, till now there is apparently no Lot-sizing rule available to find the effective solution where the entire production has to be made in some pre assigned number of lots. Even DP will fail to solve Lot sizing problem with above constraints.

This paper will present a new lot sizing heuristics that is flexible enough to take care of solving lot sizing problems where the number of lots is pre fixed. The model appears to be unique as no other known heuristic handle lot sizing problem with such condition. The proposed rule (heuristic) can be applicable to find very near optimum solution in any dynamic demand scenario. Computerization of this model has made it possible to find the instant solution just inputting necessary data.

Key words: Lot-sizing heuristics; Cost Optimization; Dynamic Programming; Wagner-Whitin algorithm

1. Introduction

There is apparently no heuristic available for solving lot sizing problems where the entire production to be made in some pre assigned number of lots. However, it is felt this type of heuristic may also prove useful under certain procurement/production scenario as listed below:

i) The true optimum solution (say by DP) may recommend too many number of lots for the entire production. Sometimes it may not be possible for management to arrange for so frequent production.

ii) The optimum solution may also recommend too less number of lots. That case a lot may cover sufficiently big number of periods which may not be good for fast consuming items like fruits and vegetables, confectionery items, medicines etc where the replenishment policy has to be optimized taking care of their shelf life because of their decaying nature

iii) Man power or machine power may be available for some fixed number of times during the entire planning horizon

iv) Shopping mall, medicine shops like retailers usually prefer to procure stationery items like food grain, oil, confectioneries etc at fixed number of times in a period (say four times in a month)

Unfortunately, even though the number of times of procurement by the retailers is fixed, they usually do at equal interval of times (may be every Sunday of a month) and quantity of procurement also as per their estimate. Most of the time, this ends up with either shortage of some items (stockout) or excess of items remaining piled on the floor. Under above circumstances, a heuristic as proposed will be very useful.

2. Literature Review

Lot sizing for dynamic demand has received considerable attention in the literature over the past two decades. A recent review and survey by DeBodt et al. (1984) cites over 200 articles on the subject.

Several researchers including Silver and Meal (1973) and Ho (1993) have compared different lot-sizing rules in scenarios under steady and also under lumpy demand conditions. But, since they have employed widely varying investigative approaches, their inferences often differ. Overall, however, the Least Period Cost (LPC) and Part Period Balancing (PPB) methods are noted to be satisfactory under varying operating factor settings. It can be difficult to compare the results obtained from different heuristics due to the different experimental settings. However, two other methods—Least Unit Cost (LUC) and McLaren's Order Moment (MOM)—show a relatively high degree of robustness across differing operating environments. Benton (1985) in his research showed that LUC method outperforms MOM, while the opposite occurs in the cases studied by Benton and Whybark (1982) and Christoph (1989). Here also comparison of the results cannot come good because of different experimental settings. When setup cost is negligible, the simple Lot-for-Lot approach provides the optimum solution. However, most lot sizing methods (rules) except Dynamic Programming are all heuristics and hence do not always guarantee optimality of the solution (Martinich, 2003).

Sphicas (2006) worked with EOQ and EPQ models using an algebraic approach to prove the formulae for EOQ and EPQ was only linear and extended the study with linear and fixed backorder cost to find condition when there should be any back ordering or not.

An approach that does guarantee an optimal solution to any non discount lot sizing problem is the Wagner-Whitin Algorithm.

Wagner and Whitin (1958) formulated the lot sizing and procurement-scheduling problem as a Dynamic Program (DP) that may be solved exactly to find the optimum lot size at different periods. However, there has been considerable controversy in the literature as to the relative advantages or disadvantages of the optimal Wagner-Whitin algorithm versus various heuristic procedures. The two basic criteria for evaluation are cost performance and computing time. As early as 1968, Tuite and Anderson (1968) investigated the performance of lot size algorithms. Berry (1972) proposed an experimental design framework for analysis along two dimensions; the coefficient of variation of demand and the ratio of ordering to inventory holding costs. A plethora of literature exists comparing new heuristic procedures and variations of these as against Wagner-Whitin algorithm [Aucamp (1985)]. Many claims about the extreme inefficiency of Wagner-Whitin were made without carefully controlled experimental designs, consideration of the use of efficient computer data structures, prior to the introduction of microcomputers. Evans (1985), Saydam and McKnew (1987), and Evans et al. (1990) recently showed that the computational performance of Wagner-Whitin can be enhanced through proper computer implementation and application of theory.
Since the advent of microcomputers, and the development of microcomputer based MRP software, the lot sizing issue has again become a subject of importance. An extensive and comprehensive computational study of the relative performance of Wagner-Whitin against several popular heuristic procedures like Economic Order Quantity (EOQ), Part-Period Balancing (PPB) [Matteis (1968)], Incremental Part-Period Algorithm (IPPA) [Boe (1983)] and Silver-Meal (SM) (1973) had been conducted by Saydam C. and James R. Evans (1990). Eight levels of time periods (N) were examined from 50 to 400 by increments of 50. The ratio of setup cost to holding cost (EPP) was varied from 125 to 500 by increments of 125 (4 levels), and the coefficient of demand variation (cv) was varied from 0.20 to 1.00 by increments of 0.20 (5 levels). This resulted in 160 experimental conditions. Ten replications of each were randomly generated and solved by each algorithm.

Average computing times (sec) taken by various models for various levels of N is shown in the following table.

Table 2.1. Average computing times in seconds

N	WW	EOQ	PPB	IPPA	SM
50	0.26	0.11	0.03	0.03	0.02
100	0.51	0.22	0.06	0.05	0.04
150	0.79	0.33	0.09	0.08	0.05
200	1.05	0.45	0.13	0.10	0.06
250	1.32	0.56	0.16	0.13	0.08
300	1.58	0.67	0.19	0.15	0.09
350	1.85	0.78	0.22	0.18	0.10
400	2.11	0.89	0.25	0.20	0.12

Table 3.1 presents average CPU times of the 10 replications for various levels of N. All times are based in seconds on a standard IBM PC, running at 4.77 MHz clock time. For example, a typical 100 period problem took 0.51 sec using WW algorithm whereas it only took 0.04 sec using SM. However, today with fast computing processors computing times can be reduced easily 20-40 times more by employing Pentium processors.

However, through literature survey we have not identified any heuristic which can solve lot sizing problems of proposed type. This has acted as the driving force to develop an algorithm what appears to be unique in nature.

3. The Proposed Heuristic for determining Lot size for regular dynamic demand with pre-assigned number of lots

The key characteristic of the proposed model is that it takes care of the variability of all problem parameters including setup cost, production cost (this variability is not considered at all in other heuristics) , holing cost and the periodic demand required for total inventory cost calculation. We shall also impose an additional constraint by assigning a fixed number of production lots and with all these we shall try to find the most cost optimized solution.

The heuristic first identifies the number of production lots and then try to divide the total number of periods in planning horizon equally (ie if planning horizon covers 12 periods and the number of lots is fixed at 4 then the heuristic will divide planning horizon in 4 equal parts each part containing 3 periods). The following steps are detailed below.

For modeling purposes, the following assumption and notations will be used:

No shortage cost

No production lot will cover periods more than 2N/n where

N Number of total periods to cover planning horizon

n Number of lots

m Number of periods covered in one lot m = 1, 2 ,3, …., 2N/n

D_i Demand for period i, i = 1, 2, 3,….., N

K_i Setup cost in period i, i = 1, 2, 3,….., N

c_i Cost of production per unit in period i, i = 1, 2, 3,….., N

h_i Holding cost per unit in period i, i = 1, 2, 3,….., N

Step 1:

Divide N into n groups, ie do N/n . If it is perfectly divisible, then all groups will have equal periods. Otherwise groups may contain different periods but should be as close as possible. (in case of 10 periods the groups may contain 2, 2, 3, 3 or 2, 3, 2, 3 periods).

Step 2:

In iteration 1, we consider first two groups. Calculate the total operating cost for both groups separately (say C_1 and C_2) as

$$C_1 = K_1 + c_1(D_1 + D_2 +\ldots+D_m) + D_2h_1+D_3(h_1+h_2)+\ldots D_m(h_1+h_2+\ldots+h_{m-1})$$

$$C_2 = K_{m+1} + c_{m+1}(D_{m+1} + D_{m+2} +\ldots+.D_{2m}) +D_{m+2}h_{m+1} + D_{m+3}(h_{m+1}+h_{m+2}) +\ldots D_{2m}(h_{m+1}+h_{m+2}+\ldots+h_{2m-1})$$

Then find the operating cost per unit for both groups separately (say C_{1u} and C_{2u}) as $C_{1u} = C_1/(D_1 + D_2 +\dots+D_m)$ and $C_{2u} = C_2/(D_{m+1} + D_{m+2} +\dots+.D_{2m})$

Case (i)

If $C_{1u} < C_{2u}$, then increase the number of periods in Group1 by shifting the first period of Group2 to the last of Group1.

Case (ii)

If $C_{1u} > C_{2u}$, then increase the number of periods in Group2 by including the last period of Group1 as the first period of Group2.

Step 3:

In iteration 2, Calculate the total operating cost for both groups (this time containing different number of periods) separately. Then find the operating cost per unit for both groups separately (new C_{1u} and C_{2u})

If $C_{1u} < C_{2u}$ still, then increase the number of periods in Group1 further by shifting the first period of Group2 to the last of Group1.

The key characteristic of the proposed model is that it takes care of the variability of all problem parameters including setup cost, production cost (this variability is not considered at all in other heuristics) , holing cost and the periodic demand required for total inventory cost calculation. We shall also impose an additional constraint by assigning a fixed number of production lots and with all these we shall try to find the most cost optimized solution.

The heuristic first identifies the number of production lots and then try to divide the total number of periods in planning horizon equally (ie if planning horizon covers 12 periods and the number of lots is fixed at 4 then the heuristic will divide planning horizon in 4 equal parts each part containing 3 periods). The following steps are detailed below.

For modeling purposes, the following assumption and notations will be used:

No shortage cost

No production lot will cover periods more than 2N/n where

N Number of total periods to cover planning horizon

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Divide N into n groups, ie do N/n . If it is perfectly divisible, then all groups will have equal periods. Otherwise groups may contain different periods but should be as close as possible. (in case of 10 periods the groups may contain 2, 2, 3, 3 or 2, 3, 2, 3 periods).

Step 2:

In iteration 1, we consider first two groups. Calculate the total operating cost for both groups separately (say C_1 and C_2) as

$C_1 = K_1 + c_1(D_1 + D_2 +\dots+D_m) + D_2h_1+D_3(h_1+h_2)+\dots D_m(h_1+h_2+\dots+h_{m-1})$

$C_2 = K_{m+1} + c_{m+1}(D_{m+1} + D_{m+2} +\dots+.D_{2m}) +D_{m+2}h_{m+1} + D_{m+3}(h_{m+1}+h_{m+2}) +\dots D_{2m}(h_{m+1}+h_{m+2}+\dots+h_{2m-1})$

Then find the operating cost per unit for both groups separately (say C_{1u} and C_{2u}) as $C_{1u} = C_1/(D_1 + D_2 + \ldots + D_m)$ and $C_{2u} = C_2/(D_{m+1} + D_{m+2} + \ldots + D_{2m})$

Case (i)

If $C_{1u} < C_{2u}$, then increase the number of periods in Group1 by shifting the first period of Group2 to the last of Group1.

Case (ii)

If $C_{1u} > C_{2u}$, then increase the number of periods in Group2 by including the last period of Group1 as the first period of Group2.

Step 3:

In iteration 2, Calculate the total operating cost for both groups (this time containing different number of periods) separately.

Then find the operating cost per unit for both groups separately (new C_{1u} and C_{2u})

If $C_{1u} < C_{2u}$ still, then increase the number of periods in Group1 further by shifting the first period of Group2 to the last of Group1.

Step 4:

The process has to be continued until

C_{1u} overtakes C_{2u} ie C_{1u} exceeds C_{2u} if it was earlier less than C_{2u} for case(i) and just opposite for case(ii). Then we stop further iteration. We calculate $C_1 + C_2$ (say C) for every iteration. The group G1 corresponding to the lowest C will determine the first Lot.

Step 5:

After the first lot has been decided, the group G2 corresponding to the lowest C (from where G1 has been considered for the first lot) will be taken with G3 and the same procedure will be followed for determining the second lot.

Step 6:

The procedure will be continued until the last lot (end of planning horizon) is met.

Numerical illustrations

The Experimental Design:

To test this heuristic, several experiments have been conducted with various Demand Size, and other associated costs (like set up, production and holding cost) and different number of lots. We shall illustrate this heuristic with one suitable example to see how easily we find the optimum solution. The result , however cannot be verified by any other lot sizing model because of their inability of solving this type of problem.

Example : A dynamic regular demand

Let us consider the following example of a regular demand for 12 periods.

Suppose the entire demand has to be met in 4 lots.

We first divide the 12 (N) periods into 4 (n) groups (G1, G2, G3 and G4). Thus each group will have 3 (m) periods. The procedure is illustrated through the following iterations.

Production of Ist Lot:

Here Total cost per unit for G1 is less than that for G2. Hence in next alternative we shall shift the period 4 to be included as the last period of G1 and find the total cost for G1 and G2. This has been shown in the Table 3.2.

Total cost per unit for G1 is still less than that for G2. Hence in next alternative we shall shift the period 5 to be included as the last period of G1 and apply the same technique. Table 3.3 shows it in details.

Table 3.1: The problem:

No.	Period	Demand	Setup Cost	PC/ unit	HC/ unit
	(i)	(D)	(K)	(c)	(h)
1	1	50	150	10	1
2	2	40	120	12	3
3	3	60	90	15	1
4	4	40	50	16	2
5	5	50	100	15	1
6	6	60	120	12	1.2
7	7	35	140	14	1
8	8	40	160	10	2
9	9	45	150	15	3
10	10	50	100	10	1.5
11	11	55	90	11	1
12	12	60	100	12	3

Production of 1st Lot

Table 3.2: Alternative 1:

Group	Period	Demand	Setup Cost	PC/ Unit	HC/ unit	Holding cost	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)			
G1	1 2 3	50 40 60	150 120 90	10 12 15	1 3 1	0 40 240	1930	12.87
G2	4 5 6	40 50 60	50 100 120	16 15 12	2 1 1.2	0 100 180	2730	18.20

Total Cost for G1 and G2 = 4660

Table 3.3: Alternative 2:

Group	Period	De- mand	Setup Cost	PC/ unit	HC/ unit	Holding cost	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)			
G1	1 2 3 4	50 40 60 40	150 120 90 50	10 12 15 16	1 3 1 2	0 40 240 200	2530	13.32
G2	5 6	50 60	100 120	15 12	1 1.2	0 60	1810	16.45

Total cost for G1 and G2 = 4340

Table 3.4: Alternative 3:

Group	Period	De-mand	Setup Cost	PC/ Unit	HC/ unit	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)		
G1	1	50	150	10	1	3380	14.08
	2	40	120	12	3		
	3	60	90	15	1		
	4	40	50	16	2		
	5	50	100	15	1		
G2	6	60	120	12	1.2	840	14.00

Total cost for G1 and G2 = 4220

Total cost per unit for G1 is now more than that for G2. No further attempt is required.
In three alternatives, the total cost for G1 and G2 is minimum in alternative3.
Hence first lot will produce demands for the periods from 1 to 5 with total operating cost of 3380

Production of 2nd Lot

Table 3.5: Alternative 1:

Group	Period	Demand	Setup Cost	PC/ Unit	HC/ unit	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)		
G2	6	60	120	12	1.2	840	14.00
G3	7	35	140	14	1	1995	16.63
	8	40	160	10	2		
	9	45	150	15	3		

Total cost for G2 and G3 = 2835

Here Total cost per unit for G2 is less than that for G3. Hence in next alternative we shall shift the period 7 to be included as the last period of G2.

Table 3.6: Alternative 2:

Group	Period	Demand	Setup Cost	PC/ unit	HC/ unit	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)		
G2	6	60	120	12	1.2	1302	13.71
	7	35	140	14	1		
G3	8	40	160	10	2	1100	12.94
	9	45	150	15	3		

Total cost for G2 and G3 = 2402
Total cost per unit for G2 has now exceeded the same for G3. No further attempt is required.

In two alternatives, the total cost for G2 and G3 is minimum in Alternative 2.

Hence second lot will produce demands for the periods from 6 and 7 with total operating cost of 1302.

Production of 3rd Lot :

Table 3.7: Alternative 1:

Group	Period	Demand	Setup Cost	PC/ unit	HC/ unit	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)		
G3	8 9	40 45	160 150	10 15	2 3	1100	12.94
G4	10 11 12	50 55 60	100 90 100	10 11 12	1.5 1 3	1982.5	12.02

Total cost for G3 and G4 = 3082.50

Here Total cost per unit for G3 is more than that for G4. Hence in next alternative we shall shift the period 9 from G3 to be included as the first period of G4 and calculate the same once again.

Table 3.8: Alternative 2:

Group	Period	Demand	Setup Cost	PC/ unit	HC/ unit	Total cost	Total cost Per unit
	(i)	(D)	(K)	(c)	(h)		
G3	8	40	160	10	2	560	13.71
G4	9 10 11 12	45 50 55 60	150 100 90 100	15 10 11 12	3 1.5 1 3	4027.5	19.18

Total cost for G3 and G4 = 4587.50

Total cost per unit for G3 has now become less than that of G4. Hence no further attempt is required.

In two iterations, the total cost for G3 and G4 is minimum in alternative 1.

Hence third lot will produce demands for the periods 8 and 9 with total operating cost of 1100.

The fourth lot will produce demands for the remaining periods from 10 to 12 with total operating cost of 1982.50.

The total operating cost will be 7764.50 .

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In such problem, if a lot size is determined as covering periods of 2 groups (say first lot of this problem covers demands of periods 1 to 6 ie demands of G1 and G2), the remaining periods will be divided in n-1 groups.

This becomes one positive feature of this heuristic that no lot can cover periods more than 2N/n.

As no heuristic has handled this kind of problem so far (maximum applications in retail business), so the comparison of the result with others' could not be possible. Incidentally the solution of the problem as solved by Dynamic Programming is exactly same as that given by the proposed heuristic.

Conclusions

Lot sizing problems are the most practical problems need to be solved quite frequently (perhaps on daily basis) at every node of the Supply Chain. This may be found particularly retail business. It is also equally important to apply the lot sizing in the procurement and the manufacturing sectors of the supply chain.

The proposed heuristics is unique in nature considering the type of lot sizing problem it can solve. The heuristic will be immensely beneficial for solving lot sizing problems for procurement stationery items in retailed business.

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