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A SINGLE ITEM MULTI PERIOD LOT-SIZING RULE FOR TOTAL OPERATING COST OPTIMIZATION WITH INCREMENTAL DISCOUNT ON PRODUCTION QUANTITY

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ABSTRACT

Material Requirements Planning (MRP) characteristically involves lot sizing decisions to be made when demand is both deterministic and dynamic (regular as well as lumpy) and spread over a finite time horizon. Over past four decades a number of heuristic methods have been developed to find optimum lot sizes that minimize the total (inventory and production) cost. In the restricted case when demand is stable and known over the decision horizon, the simple static EOQ model can find the optimum solution. The exact solution in more general situations, assuming integral-valued demand, is obtainable—by Dynamic Programming (DP). This paper presents a new lot sizing heuristic that can be a good substitute for the DP method. This method is simpler and it applies to both continuous and lumpy demand conditions. ***The method also overcomes several limitations of other conventional methods like Part period balancing, Silver & Meal, Least unit cost etc that use cost minimization as the objective in determining lot sizes.*** *Specifically, the proposed method permits time-dependent variations in problem parameters such as setup cost, unit production cost, and unit holding cost while its computational performance remains almost unaffected by such variations.*

Introduction

In many production planning or procurement scenarios, strictly static demand is rarely encountered. However, if demand is not constant over a period of time, the simple EOQ model cannot correctly minimize the total operating costs. Consequently, several different heuristic methods have been devised to find lot sizes that minimize the total cost (inventory and production) in the dynamic demand scenario. Still, cost factors such as setup, unit holding cost, etc. may also vary with time. This paper proposes a new heuristic for solving such problems. We demonstrate its simplicity and robustness and then compare its performance with the prevailing lot sizing rules used in MRP.

Optimal lot sizing is always viewed as a major challenge in the context of supply chain management. Several heuristics have been developed to optimize lot-size so as to minimize the total materials management cost. However, these methods often suffer from practical limitations and do well only under particular conditions. For instance, the conventional static EOQ approach to determine the lot size overlooks variations in requirements at different periods. A simpler method like Lot-for-Lot may do well where set up cost is less significant. But its

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performance degrades significantly with high setup costs, or in lumpy demand conditions—a phenomenon common in MRP. Another heuristic, the Part Period Balancing method, does well under lumpy demand conditions, but it assumes that setup cost and holding costs remain unchanged over the periods, a provision not always true. Still other lot sizing heuristics such as the Silver-Meal (SM), or Least Unit Cost (LUC) also attempt to minimize the inventory cost under certain conditions. But these methods do not guarantee that the resulting total cost will always be minimum.

Lot sizing for dynamic demand has received considerable attention in the literature over the past two decades. A recent review and survey by DeBodt et al. (1984) cites over 200 articles on the subject.

Several researchers including Silver and Meal (1973) and Ho (1993) have compared different lot-sizing rules in scenarios under steady and also under lumpy demand conditions. But, since they have employed widely varying investigative approaches, their inferences often differ. Overall, however, the Least Period Cost (LPC) and Part Period Balancing (PPB) methods are noted to be satisfactory under varying operating factor settings. It can be difficult to compare the results obtained from different heuristics due to the different experimental settings. However, two other methods—Least Unit Cost (LUC) and McLaren's Order Moment (MOM)—show a relatively high degree of robustness across differing operating environments. Benton (1985) in his research showed that LUC method outperforms MOM, while the opposite occurs in the cases studied by Benton and Whybark (1982) and Christoph (1989). Here also comparison of the results cannot come good because of different experimental settings. When setup cost is negligible, the simple Lot-for-Lot approach provides the optimum solution. However, most lot sizing methods (rules) except Dynamic Programming are all heuristics and hence do not always guarantee optimality of the solution (Martinich, 2003 **).**

Sphicas (2006) worked with EOQ and EPQ models using an algebraic approach to prove the formulae for EOQ and EPQ was only linear and extended the study with linear and fixed backorder cost to find condition when there should be any back ordering or not.

However, all these rules are just heuristics and do notnot give any guarantee of offering optimum solution and sometimes result may depart too much from the true solution.

Only one approach that does guarantee an optimal solution to any non discount lot sizing problem is the **Wagner-Whitin Algorithm.**

Wagner and Whitin (1958) formulated the lot sizing and procurement-scheduling problem as a Dynamic Program (DP) that may be solved exactly to find the optimum lot size at different periods.

However, this algorithm could not become popular for at least two reasons. First, the unlike EOQ or lot-for-lot, the DP methodology is not commonly known to the majority of MRP practitioners. Secondly, the calculations that determine optimum lot size by DP are cumbersome even for a moderate planning period. To simplify computation, even computerized DP models assume only integer-valued demand quantities and that too to be within a limit. Such limitations along with the method's inherent complexity to find solutions has made DP-based lot sizing not so widespread in industry, even if it provides true optimality.

An extensive and comprehensive computational study of the relative performance of Wagner-Whitin against several popular heuristic procedures like Economic Order Quantity (EOQ), Part-Period Balancing (PPB), Incremental Part-Period Algorithm (IPPA) and Silver-Meal (SM) had been conducted by Saydam C. and James R. Evans (1990). The study found out the

Average computing times (sec) taken by various models for various levels of time periods (N) as shown in Table 1.

Table 1: Average computing times in seconds

N	WW	EOQ	PPB	IPPA	SM
50	0.26	0.11	0.03	0.03	0.02
100	0.51	0.22	0.06	0.05	0.04
150	0.79	0.33	0.09	0.08	0.05
200	1.05	0.45	0.13	0.10	0.06
250	1.32	0.56	0.16	0.13	0.08
300	1.58	0.67	0.19	0.15	0.09
350	1.85	0.78	0.22	0.18	0.10
400	2.11	0.89	0.25	0.20	0.12

Zhao (1998) compares the performance of a new improved heuristic (STIL) proposed by Coleman and McKnew (1991) against those of the Wagner-Whitin (WW), cost-modified Wagner-Whitin (MWW) and some simple lot-sizing rules on the selection of the parameters for freezing the master production schedule (MPS). The experimental settings include forecasting, master production scheduling and material requirements planning under a rolling time horizon. The result of the study shows that the STIL heuristic, the WW rule and the MWW rule do not really outperform the simple cost-modified Silver-Meal and other heuristics even though they require substantially higher computation time. The study also shows that the selection of the parameters for freezing the MPS is not significantly influenced by the selection of the lot-sizing rules.

Huan et al (2005) solved the problem of incorporating both learning and forgetting in setups and production into the dynamic lot-sizing model to obtain and optimal production policy, including the optimal number of production runs and the optimal production quantities during the finite period planning horizon.

The study shows that no effective heuristic has been developed so far to handle lot sizing problem with incremental discount in production quantity, a very practical consideration in any production scenario. This paper presents a heuristic method discussed in the following section. The method is illustrated with one numerical example followed by conclusion.

The Proposed Heuristic

In normal production scenario, the production cost is not strictly proportional to the production quantity. This heuristic which will consider incremental discount, i.e. some discount in unit production cost if the production quantity exceeds certain limit(s) (there can be several levels of production quantity on which different discount rates can be offered) is in the process of completion

The key characteristic of the proposed model is that it takes care of the variability of all problem parameters including setup cost, production cost (this variability is not considered at all in other heuristics), holing cost and the periodic demand required for total inventory cost calculation.

The heuristic identifies the successive future periods whose demand can be filled through current period's production, and while doing so it considers the monetary opportunity loss (or gain) when the requirement(s) for the following periods are considered to be produced in the current period.

For Modeling Purpose, the following Assumptions and Notations will be used:

No shortage cost

No limit on production capacity

No limit on storage capacity

Demand of a period cannot be divided in two separate lots

No inventory holding cost if the demand is met in the same period

N	Number of periods
n	Number of remaining periods in each iteration (n <= N)
D_i	Demand for period i, i = 1, 2, 3,….., N
K_i	Setup cost in period i, i = 1, 2, 3,….., N
c_i	Cost of production per unit in period i, i = 1, 2, 3,…..,
h_i	Holding cost per unit in period i, i = 1, 2, 3,….., N
g_i	Effective gain in period i, i = 1, 2, 3,….., N
G_i	Cumulative gain in period i, i = 1, 2, 3,….., N
d_i	Discount (%) offered for quantity level Q_i

Assuming the absence of any initial inventory, the demand for period i = 1 has to be produced in period 1 always. Now if we produce the demand for period 2 (D_2) also in period 1, then we come across the following additional costs.

Holding Cost

It will incur an additional cost of D_2h_1 when the demand D2 is being produced at period 1, so it will be a negative gain. (= - D_2h_1)

Production Opportunity Cost

Now if c_2< $c1_{i,}$ then we shall also lose the opportunity of producing at less cost by an amount D_2 (c_1-c_2), if we produce D_2in the period 1., or in other words we gain by - $D_2(c_1-c_2)$

Setup Cost

This is negative cost. In fact we shall gain by avoiding the set up cost K2 if we produce D_2 in the period 1 by an amount K2.

Total Gain

Hence production of D_{i+1} in the period i results in the total gain as

$$\mathbf{g_2= -[\ D_2 h_1 + D_2 (c_1-c_2) - K_2]}$$

This g has to be calculated for each successive period starting from the second period of each iteration (as g_1 is always zero).

Cumulative Gain CGn = $g_2 + g_3 + \dots + g_n$

Discount Opportunity Gain

Let us assume there are two discounts d_1 and d_2 to be offered for two quantity levels Q_1 and Q_2 respectively.

If $+D_i < Q_1$ (Q1 is the quantity level for first discount offer) then no discount will be available

If $Q1 < +D_i < Q_2$ where Q_2 is the quantity level for second discount offer, then

Discounted amount $Ai = (+D_i - Q_1)\, d_1$

If $+D_i > Q_2$ then

Discounted amount $Ai = (+D_i - Q_2)\, d_2 + (Q_2 - Q_1)\, d_1$ and so on.

A_i has to be calculated for periods starting from 1 to n ($A1, A_2, A_3, \ldots A_n$) in each iteration.

Total cumulative gain with discount for a lot covering n periods in an iteration will be $Gn = CG_n + A_n$

Determining Lot size (1st Lot)

Suppose period 2 demand ($D_{2)}$ is being produced at period 1

That case, if g_2 is positive (i.e. actually a gain) and more than the corresponding setup cost K_2, then **it can be proved that it is always profitable to produce of D_2 in the period 1** (proof is given later). If g2 is negative then it will always incur loss to produce **D_2 in the period 1**, and hence **D_2 need not be produced in period 1**.

But if g_2 is positive but less than K_2, the cumulative gain per unit product will be considered for determining the exact lot size.

The rule is applicable for all successive periods.

The Lot size (n) will be determined by the conditions below:

We include the demands of the periods to be produced in first lot so long $G_n/+D_n$ value shows the decreasing trend.

When **$G_{n+1}/+D_{n+1} < G_n/+D_n$** we check if also $g_{n+1} < K_{n+1.}$

If both conditions are satisfied then the first lot will cover first n periods for production.

That means we shall stop at the period where **$G_{n+1}/+D_{n+1} < G_n/+D_n$** and also **$g_{i+1} < K_{i+1}$** .

Proof

Suppose

D_1 and D_2 are the demands for period 1 and period 2

K_1 and K_2 are the Setup costs for period 1 and period 2

c_1 and c_2 are the Unit production costs for period 1 and period 2

h_1 and h_2 are the Unit holding costs for period 1 and period 2

Now if D_2 demand is produced in period 1 then

Overall gain $g_2 = K_2 - D_2\,(c_1 - c_2) - D_2 h_1$ {From (i)}

Now if $g_2 > K_{2,}$ then $c_1 + h_1 - c_2 < 0$

Cost of producing D_2 in period 1, $C_1 = D_2 c_1 + D_2 h_1$

Cost of producing D_2 in period 2, $C_2 = D_2c_2 + K_2$

$C_1 - C_2 = D_2(c_1 + h_1 - c_2) - K_2$ ie always less than zero.

Hence $C_1 < C_2$.

The Steps of this Heuristic are

Step 1: Set i = 1 (iteration 1)

Step 2: Go to the period i = 2.

Calculate the total opportunity gain $g_2 = -[\, D_2h_1 + D_2\,(c_1\text{-}c_2) - K_2\,]$, Cumulative gain CG_2 (= g1 + g2), Cumulative gain with discount (A_2) Total Cumulative gain for 2 periods $G_2 = CG_2 + A_2$.

Then we calculate Total Cumulative gain per period $G_2/2$ and

Total Cumulative gain per product $G_2/(D_1 + D_2)$

If $G_2/2<G_1/1$ and also $\mathbf{G_2/(D_1 + D_2) < G_1/D_1}$ **,** then stop production for 2nd period & hence the first lot will produce for the first period only. Otherwise go to Step 3.

Step 3: Go to the period i = 3

Calculate the total opportunity gain $g_3 = -[\, D_3(h_1+h_2) + D_3\,(c_1\text{-}c_3) - K_3\,]$,

Cumulative gain CG_3 ($=g_1+g_2+g_3$) and Cumulative gain with discount (A_3).

Total Cumulative gain for 3 periods $G_3 = CG_3 + A_3$.

Then we calculate Total Cumulative gain per period $G_2/3$ and

Total Cumulative gain per product $G_3/(D_1 + D_2 + D_3)$

If $G_3/3<G_2/2$ and also $\mathbf{G_3/(D_1 + D_2 + D_3) < G_2/(D_1 + D_2)}$**,** then stop production for 2nd period & hence the first lot will produce for the first and second period.

Otherwise we proceed to consider the further periods 4, 5 etc., until the both conditions are fulfilled for considering the first lot production.

Step 4: (iteration 2)

Set i = n+1 (n is the number of periods covered in the first period) and continue to repeat the same procedure as stated in step 2 and step 3.

If i >N, then stop, the entire planning horizon has been covered.

Numerical Illustrations

Example: A Regular and Dynamic Demand

Let us apply the heuristic to find the solution of a Lot sizing problem with 12-period dynamic but deterministic demand. While solving the problem, the heuristic will take care variations of all parameters along with the Discount opportunity offered on the production quantity. The rule starts with calculating the following opportunity losses.

HC: Holding Cost for producing demand for future period(s) in the present period, which will be always positive

POC: Production Opportunity Cost (or loss). This loss is due to producing demand for future period in the present period and this can be positive or negative depending on whether present period unit production cost is more or less than that in future period.

SOC: Setup Opportunity Cost. This is due to cost saving by avoiding setup cost of future period(s) by producing demands for future period(s) in the current period , and hence it will be always negative i.e. always a gain.

Then we calculate

i) Total effective gain g = - (HC + POC + SOC)

ii) Cumulative gain CG = $g_1 + g_2 + g_3$ +.....

iii) Discount opportunity gain A (by formula already explained)

iv) Cumulative gain with discount G = CG + A

At last average gain per unit product (G**/+D**) as per formulae already discussed will be calculated for fixing the lot size.

At period *i*=4, G/ΣD maximum and then it falls in the next period.

Table 2: 1st Lot Production

No.	Period	Demand	Setup Cost	PC/ unit	HC/ unit	Various Opportunity loss			Total Gain	Cum Gain	Disc Gain	Total Cum Gain	Gain Per Unit
(n)	(i)	(D)	(K)	(c)	(h)	HC	POC	SOC	g	CG	A	G = CG+A	G/ΣD
1	1	50	40	100	1	0	0	0	0	0	0	0	0
2	2	80	60	120	1.6	80	-1600	-60	1580	1580	0	1580	12.15
3	3	60	90	115	1	156	-900	-90	834	2414	0	2414	12.71
4	4*	40	80	108	3	144	-320	-80	256	2670	300	2970	12.91
5	5	100	100	100	1	660	0	-100	-560	2110	1300	3410	10.33
6	6	60	60	120	1.2								
7	7	35	70	125	1								
8	8	40	80	160	2								
9	9	45	90	112	3								
10	10	50	50	90	1.5								
11	11	55	55	110	1								
12	12	60	60	100	3								

Also in the next period (ie i = 5) $g_5 < K_5$.

Hence first lot production will take care of the demands for periods 1, 2, 3 and 4.

Total cost for first lot production is calculated as below:

TC = 40 + 100*(50 +80 + 60 + 40) + 80 + 156 + 144 – 300 = 23120 to produce 230 units

Interesting to see that though G/ΣD value falls from period 8 to period 9 but at period i=9, $g_9 > K_9$, so as per our hypothesis, 9th period demand should be included in the second lot production.

However, G/+D value further falls from period 9 to period 10 but this time $g_{10} < K_{10}$ So 10th period demand cannot be considered in the said lot.

So second lot production will produce the demands for periods from 5 to 9.

Total cost for 2nd lot production will be

= 100 + 100*(100 +60 + 35 + 40 + 45) + 60 + 77 + 128 + 234 – 800 = 27799 to produce 280 units .

Here also G/ΣD drops from period 11 to period 12 but at period i=12, $g_{12} > K_{12}$, so as per our hypothesis, 12^{th} period demand should be included in the third lot production.

Table 3: (2nd Lot Production)

Period	Demand	Setup Cost	PC/ unit	HC/ unit	Various Opportunity loss			Total Gain	Cum Gain	Disc Gain	Total Cum Gain	Gain Per Unit
(i)	(D)	(K)	(c)	(h)	HC	POC	SOC	g	CG	A	G = CG+A	G/ΣD
5	100	100	100	1	0	0	0	0	0	0	0	0
6	60	60	120	1.2	60	-1200	-60	1200	1200	0	1200	7.50
7	35	70	125	1	77	-875	-70	868	2068	0	2068	10.61
8	40	80	160	2	128	-2400	-80	2352	4420	350	4770	20.30
9*	45	90	112	3	234	-540	-90	396	4816	800	5616	20.06
10	50	50	90	1.5	410	500	-50	-860	3956	1170	5126	15.53
11	55	55	110	1								
12	60	60	100	3								

Table 4: (3rd Lot Production)

Period	Demand	Setup Cost	PC/ unit	HC/ unit	Various Opportunity loss			Total Gain	Cum Gain	Disc Gain	Total Cum Gain	Gain Per Unit
(i)	(D)	(K)	(c)	(h)	HC	POC	SOC	g	CG	G/n	G = CG+A	G/ΣD
10	50	50	90	1.5	0	0	0	0	0	0	0	0
11	55	55	110	1	82.5	-1100	-55	1072.50	1072.50	0	1072.50	10.21
12	60	60	100	3	150	-600	-60	510	1582.50	0	1582.50	9.59

Hence third lot production will take care of the demands for periods 10, 11 and 12.

Total cost for 3rd lot production will be

= 50 + 90*(50 + 55 + 60) + 82.5 + 150 – 0 = 15132.50 to produce 165 units.

Total Operating Cost of Complete Production = 66051.50 to produce 675 units.

Table 5: Comparison of Results Obtained using other Models

Model	Total Cost Without discount	Deviation from Proposed model
The Proposed model	**67151.50**	--
Dynamic programming (DP)	67151.50	No difference
Silver Meal Heuristic (SM)	74392.00	10.78 % more
Lot for Lot (L4L)	76220.00	13.50 % more
Least Unit Cost (LUC)	74420.00	10.82 % more
Least Total Cost (LTC)	74040.00	10.26 % more
Part Period Balancing (PPB)	74040.00	10.26 % more
Dynamic programming (DP)	67151.50	No difference

Comparison of Results Obtained using other Models

In fact, no comparison possible as no heuristic along with DP does discount analysis. However, this model is also capable of solving the problem without discount analysis. Total cost without discount (obtained by adding the discount amount 1100 to the optimum value given by the proposed model) can then be used for comparison.

The table below (6.1.4) shows the optimum solution obtained from the proposed model without discount is even better than that obtained from all other heuristics and just matching the solution offered by dynamic programming.

Conclusion

Lot sizing problems are the most practical problems need to be solved quite frequently (perhaps on daily basis) at every node of the Supply Chain. This may be found in FMCGs and in particular retail business. It is also equally important to apply the lot sizing in the procurement and the manufacturing sectors of the supply chain.

1. The heuristics proposed in this thesis showed that they are computationally efficient in solving the practical problems in the supply Chain Management.
2. The heuristics can handle the incremental discount on production quantity which is otherwise not possible by Dynamic Programming and other heuristic methods in the literature.

In case of very high cost of production and holding cost the proposed model is flexible enough to incorporate the factor like time value of money. Also the following conditions can also be added while considering the future scope for up gradation the model:

- Limitation in storage capacity
- Limitation in production capacity
- Lot size optimization for multi products
- Maximum number of periods in one lot (this may be useful for short living items

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