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GLOBAL FINANCIAL SUPPLY CHAINS – AN INTEGER PROGRAMMING FORMULATION

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ABSTRACT

Most of the research work on supply chain management has been the study of materials flow and very little work has been done on the study of upstream flow of money. In this paper we study the flow of money in a supply chain from the viewpoint of a wholesaler who is part of a global supply chain in which the financial transactions are done in two different currencies. The wholesaler receives money from the downstream partners (retailers) and makes payments to the upstream partners (manufacturers). The objective of the wholesaler is to schedule the payments to the manufacturers within the constraints of the receipt of the money from the retailers. The wholesaler has to pay a penalty if the payments to the manufacturers are not made within the specified time. Any unused money in a given period can be invested to earn an interest. We develop an integer programming model to maximize the net cash value at the end of the operational cycle.

Introduction

Supply-chain management revolves around coordination and cooperation among several business partners that are linked through flows of material, money and information. These partners include suppliers of basic raw materials and component parts, manufacturers, wholesalers, distributors, transporters, retailers, banks and financial institutions. In general the materials, component parts and finished goods flow downstream and the money flows upstream in a supply chain. The information flows in both directions.

Most of the research work on supply chain management has been the study of material flow and very little work has been done on the study of upstream flow of money. In fact we are not aware of any research work that focuses on building an optimization model of money flows in a supply chain similar to the models that are available for material flow. In this paper we focus on the study of flow of money in a supply chain and develop an integer programming model for optimization. The formulation in this paper draws on the concepts provided in Manoj et. al. (2007) for study of material flow.

We consider the problem from the viewpoint of a wholesaler who receives finished products from manufacturers (upstream partners) and then distributes the finished products to the retailers (downstream partners). The wholesaler receives money from the downstream partners and makes payments to the upstream partners. The wholesaler is part of a global supply chain in which the financial transactions are done in two different currencies. The objective of the wholesaler is to schedule the payments to the manufacturers within the constraints of the receipt of the money from the retailers. The wholesaler has to pay a penalty if the payments to

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the manufacturers are not made within the specified time. Any unused money in a given period can be invested to earn an interest. We develop an integer programming model to maximize the net cash value at the end of the operational cycle. All monetary transactions take place through the wholesaler's bank.

Problem Scenario

The planning period, called the operational cycle in this paper, consists of *n* time periods of equal lengths *t*. There are *m* retailers and *n* (equal to the number of time periods) manufacturers. The retailers place orders with sufficient lead times so that delivery of products to the retailers is completed at the beginning of the operational cycle. However, the flow of money, which is the subject matter of this paper, continues over the entire operational cycle.

All payments, from retailers to the wholesaler and from the wholesaler to the manufacturers are due at the end of period 1. However, wholesaler runs the risk of delinquencies and may get into cash flow problem if the payments from the retailers are not received in time. To solve the cash flow problem and to reduce the financial risk, the wholesaler enters into an agreement with his bank according to which the bank agrees to pay a specified amount of money to the wholesaler at the end of every period and the bank in turn collects the money from the retailers. The bank negotiates the term of payments with individual retailers. The number of retailers, currency of payment by a retailer to the bank, the time of payment and amount of payment do not impact the model formulation in this paper. The retailers may even use the local currencies to pay to the bank. These financial transactions between the retailers and the bank, though important, are not within the scope of this paper.

The monetary transactions take place in two currencies C_1 and C_2. However, for sake of simplicity in model formulation, we will represent the amounts of the two currencies in equivalent units of a common currency say U.S. dollars ($). Let the exchange rates be ì$_1$ and ì$_2$ between ($) and the two currencies C_j *(j =1,2)*. Let δ_{ju} be the amount of currency C_j *(j= 1,2)* that has to be paid to manufacturer M_u *(u = 1,2,…., n)*; and β_{jw} be the amount of currency C_j *(j= 1,2)* that retailer R_w = *(w = 1, 2,…..m)* will pay to the wholesaler. The equivalent amounts of monetary transactions in ($) will then be $d_{ju} = \delta_{ju}\,\mu_j$ and $b_{jw} = \beta_{jw}\,\mu_j$. Let Q_u be the total amount in ($) to be paid to manufacturer M_u *(u = 1,2…,n)*, where, $Q_u = d_{1u} + d_{2u}$. Similarly, let $V_w = b_{1w} + b_{2w}$ be the amount in ($) that has to be paid by retailer *w* to the wholesaler. The total flow of currency *j* in ($) is D_j, where,

$$D_j = \sum_{u=1}^{u=n} d_{ju} = \sum_{w=1}^{w=m} b_{jw} \quad j=1,2 \qquad (1)$$

In other words the total amount of currency C_j to be received from all retailers is equal to the total amount of currency C_j to be paid to the manufacturers in the operational cycle, *j = 1,2.*

The money received from the bank in every time period is based on the total flow of the two currencies, D_j, *j = 1, 2.* The wholesaler and the bank agree that the bank will make available in every time period *t (t = 1,2,…..n)* an amount that is equal to the average requirement of each currency. Let p_j be the average requirement for currency *j*, where, $p_j = D_j/n$, *j =1,2*. It may be noted that p_j is the amount in ($) because D_j is also specified in ($) as discussed above. Let $Q = p_1 + p_2$. The bank will, therefore, pay to the wholesaler an amount ($) *Q* in every time period. The total amount paid by the bank, $\psi = nQ$. It may also be noted that

$$\psi = nQ = \sum_{u=1}^{u=n} Q_u = \sum_{w=1}^{w=m} V_w \tag{2}$$

The wholesaler and the bank could have entered into an agreement in which the bank pays the entire amount at the end of period 1 that the wholesaler could use to pay to the manufacturers. In such a scenario, the bank takes the entire risk of non-payment by the retailers. In the current arrangement, the bank and the wholesaler are sharing the risk according to the negotiated contract between the two. Risk sharing is an important consideration in design of supply chains.

Under the current arrangement, the wholesaler has to schedule the payments to the manufacturers since the entire money is not available at the end of period *1*, In other words, the wholesaler has to find a sequence $V = \{v(1), v(2), \ldots v(s).. v(n)\}$, where $v(s)$ is the manufacturer number who is paid in time period s, $(s = 1,2,\ldots,n)$. We assume that all financial transactions take place at the end of each time period. Therefore, the wholesaler has to make payments to all manufacturers within the operational cycle and one and only one manufacturer will be paid in each period.

As we will discuss in Section 4, the wholesaler incurs various costs as well as earns revenue based on inventory of money. The objective of the wholesaler is to find the best sequence V^* that maximizes the total inventory of money at the end of the operational cycle.

Money Flows and Levels

Let $I_{j(s-1)}$ denote the amount of currency j (in equivalent \$) at the beginning of period s and I_{js} (in equivalent \$) be the amount at the end of period s. It may be noted that all amounts of inventory of money for both currencies are stated in equivalent units of (\$). However, from this point onwards a specific mention of the term "in equivalent \$" will be omitted. It may be noted that I_{js} is also the beginning amount of period $(s+1)$. The amount at the beginning of period *1* is, therefore, I_{j0}, $j = 1,2$. In period *1*, p_j units of currency j are deposited by the bank in the wholesaler's account that is maintained for currency j. Similarly, an amount of $d_{j,v(1)}$ units of currency j are withdrawn by the bank from the wholesaler's account for currency j to pay to the manufacturer paid in position 1. The ending amount of currency j $(j = 1,2)$ at the end of period *1* will be $I_{j1} = I_{j0} + p_j - d_{j,v(1)}$. Similarly, the ending amount of currency j $(j = 1,2)$ at the end of period 2 will be $I_{j2} = I_{j1} + p_j - d_{j,v(2)} = I_{j0} + 2p_j - d_{j,v(1)} - d_{j,v(2)}$. In general for any time period k, I_{jk} is given by the following equation.

$$I_{jk} = I_{j(k-1)} + p_j - d_{j,v(k)} = I_{j0} + k.p_j - \sum_{s=1}^{s=k} d_{j,v(s)} . \tag{3}$$

By proceeding in this way, for period n, the ending amount of currency j $(j = 1,2)$ will be:

$$I_{jn} = I_{j(n-1)} + p_j - d_{j,v(n)} = I_{j0} + n.p_j - \sum_{s=1}^{s=n} d_{j,v(s)} . \tag{4}$$

It may be noted that $I_{jn} = I_{j0}$ since

$$n.p_j = \sum_{s=1}^{s=n} d_{j,v(s)} \quad (5)$$

Costs and Revenue

In this section we discuss various costs and revenue generated from cash holdings and payment decisions.

Cost of Beginning Inventory of Cash

The beginning (initial) inventory of cash is required to ensure that the wholesaler does not run into cash flow problems because the bank provides money at a uniform rate every period whereas the amounts to be paid to the manufacturers fluctuate. Let a_j $(j = 1,2)$ be the interest rate for borrowing money (cost of capital or opportunity cost) for time period *t* per unit of cash for the two currencies respectively. Therefore, the total cost, A_j, of holding and using the beginning inventory of cash during the operational cycle, assuming a simple interest rate, is $A_j = n.a_j I_{j0}$, $j = 1,2$. The total cost for both the currencies, *A*, is:

$$A = \sum_{j=1}^{j=2} A_j$$

Interest Earned on the Accumulated Cash

As money is received from the bank and payments are made to the manufactures, there may be unused cash in a period that may be invested to earn revenue. Suppose the wholesaler earns an interest on the accumulated cash at the rate of h_1 and h_2 for time period *t* per unit of cash for the two currencies respectively. The interest earned in any period *s* is calculated based on the average amount of cash at the beginning and end of the period *s*, $(s = 1,2,..n)$. Let H_{js} be the total interest earned for currency *j* in period *s*, where, $H_{js} = (h_j).(½)[I_{j(s-1)}+I_{js}]$, $s = 1,2,…..n$. The calculation of I_{js} has been discussed earlier. Let the total interest earned for currency *j* from period *1* to *n* be denoted as $ð_j$. The value of $ð_j$ is given by equation 6.

$$\pi_j = \sum_{s=1}^{s=n} (½)\,(h_j).\,\{[I_{j(s-1)}+I_{js}]\} = (½).h_j\,.(I_{j,0} + I_{j,n}) + h_j \sum_{s=1}^{s=n-1} I_{j,s} \quad (6)$$

However, $I_{j,n} = I_{j,0}$ as discussed above. Therefore equation (6) can be written as:

$$\pi_j = h_j \sum_{s=1}^{s=n1} I_{j,s} \quad (7)$$

It may be noted that the beginning inventory of cash has a borrowing cost associated with it as discussed in the previous section. This cash, however, is considered as part of the cash flow inventory and will be reinvested. Further, the rate of borrowing money is greater than the interest earned, that is, $a_j > h_j$. Otherwise, the wholesaler will be in a happy situation of borrowing a large sum of money and invest it at a higher interest rate.

Penalty for Late Payment to the Manufacturers

As stated earlier, the wholesaler has to pay an amount d_{ju} to the manufacturer M_u *(u = 1,2,….,n)* at the end of period 1 for currency *j*, *j =1,2.* In case of a default, there is a penalty at the rate of *q%* per period. We are assuming the same penalty rate for the two currencies. The wholesaler determines and implements a sequence *V = {v(1), v(2), …v(s).. v(n)}*, where *v(s)* is the manufacturer number who is paid in time period *s* *(s =1,2,…,n)*. Thus if the manufacturer *u* is paid at the end of period s, then the total amount paid to this manufacturer is $d_{ju} + d_{ju}q(s-1)$ for currency *j*, *j =1,2.* In other words, the penalty is $d_{ju}.q(s-1)$. Let the total penalty for currency *j* be B_j, where,

$$B_j \quad = \quad \sum_{s=1}^{s=n} d_{jv(s)}.q\ (s-1) \qquad (8)$$

The total penalty, *B*, for both currencies is

$$B = \sum_{j=1}^{j=2} B_j \qquad (9)$$

Integer Program Formulation

In this section we give the integer program formulation to solve the wholesaler's problem. The total amount of cash received from the bank by the wholesaler, ψ, is equal to the total amount that has to be paid to the manufacturers. However, the receipts and payments may not match and the wholesaler incurs costs and earns revenue as discussed above. The wholesaler starts with a beginning inventory of cash, I_{j0} and ends the operational cycle with an ending inventory of cash Φ_j where,

$$\Phi_j \quad = \quad I_{j0} + \pi_j - A_j - B_{j,} \quad j = 1,2.$$

In the above expression, $\{\pi_j - A_j - B_j\}$ represents the net gain (or loss) during the operational cycle for currency *j =1,2.* Let *Đ* be the total gain (or loss) for both currencies where,

$$\Pi = \quad \sum_{j=1}^{j=2} (\pi_j - A_j - B_j)$$

The objective of the wholesaler is, therefore, to maximize the net cash value, Π, at the end of the operational cycle.

The following mixed integer program formulation will maximize the objective function for this problem. Let $x_{us} = 1$ if the manufacturer *u* is paid in period *s*. Therefore, the objective function and constraints can be written as:

$$\text{Maximize} \qquad \Pi \quad = \quad \sum_{j=1}^{j=2} h_j \sum_{s=1}^{s=n} I_j, s \ \ - \ \sum_{j=1}^{j=2} n.a_j I_{j0} \ \ - \ \sum_{j=1}^{j=2} \ \sum_{s=1}^{s=n} \{ d_{ju}.\ x_{us}.\ q\ (s-1) \qquad (8)$$

Subject to:

$$\sum_{u=1}^{u=n} x_{us} = 1,\ s = 1,2,\dots,n \qquad (9)$$

$$\sum_{s=1}^{s=n} x_{us} = 1,\ u = 1,2,\ldots,n \qquad (10)$$

$$I_{1s} = I_{1,s-1} + p_1 - \sum_{u=1}^{u=n} d_{1u}.x_{us}, \qquad s = 1,2,,,,n \qquad (11)$$

$$I_{2s} = I_{2,s-1} + p_2 - \sum_{u=1}^{u=n} d_{2u}.x_{us}, \qquad s = 1,2,,,,n \qquad (12)$$

$$I_{1n} = I_{10} \qquad (13)$$

$$I_{2n} = I_{20} \qquad (14)$$

I_{1s} e≥ 0, s = 1,….., n (15)

I_{2s} e” 0, s = 1,….., n (16)

x_{us} º {0,1), s = 1,…, n; u = 1,…..,n. (17)

Equations (9) and (10) represent the constraints that one and only one manufacturer is paid in each period. Equations (11) and (12) specify the constraints to balance the cash at the beginning and end of each period. Equations (13) and (14) enforce the constraints that ending cash is equal to the cash at the beginning of the operational cycle. Equations (15), (16) represent the non-negativity constraints and equation (17) forces x_{us} to take values of 0 and 1 only.

Conclusions and Discussions

This paper is probably the first attempt to model the global financial supply chains. We consider the problem of upstream cash flow of two currencies in a global environment and provide an integer programming formulation of the problem. In future research the formulation can be extended to multiple currencies. We have assumed in this paper that the exchange rate remains constant over the operational cycle and only one manufacturer will be paid in each period. These assumptions also need to be relaxed in future research on this topic.

References

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